MATHEMATICS

Sol. 1 (C)

$3^{1/3}$	$7^{1/7}$	1
0	0	10
3	0	7
6	0	4
9	0	1
3	7	0
0	7	3

$\therefore$ no. of terms are 6

Sol. 2 (B)

Given that $T_5 + T_6 = 0$

$^nC_4\, a^{n-4} (-b)^4 + {}^nC_5\, a^{n-5} (-b)^5 = 0$

$\Rightarrow a^{n-5}b^4 [{}^nC_4\, a - {}^nC_5 b] = 0$

$\Rightarrow {}^nC_4 a = {}^nC_5 b \quad (\because a \neq 0, b \neq 0)$

$$\Rightarrow \frac{a}{b} = \frac{^nC_5}{^nC_4} = \frac{n-4}{5}$$

Sol. 3 (D)

Origin lies left to the line. Points (2, 3/4) & (1/4, −1/4) lie in the smaller part & also in the circle so only two points.

Sol. 4 (A)

$$\frac{z_1}{r_1} = \frac{z}{r} = e^{i\pi}$$

$$\frac{z_1}{3r} = -\frac{-z}{r}$$

$z_1 = -3z = -3(4 - 3i)$

$z_1 = -12 + 9i$

Sol. 5 (A)

$\bar{z} z^3 + z \bar{z}^3 = 350$

$z\bar{z}(\bar{z}^2 + z^2) = 350$

Put $z = x + iy$

$(x^2 + y^2)(x^2 - y^2) = 175$

$(x^2 + y^2)(x^2 - y^2) = 5.5.7$

$x^2 + y^2 = 25$

$x^2 - y^2 = 7$

$x = \pm 4,\ y = \pm 3$

$x, y \in I$

Area = $8 \times 6 = 48$ sq. units

Sol. 6 (C)

common diff. = d, in A.P.

$T_7 = 9 \Rightarrow a + 6d = 9 \Rightarrow a = (9 - 6d)$

$T_1T_2T_7 = a\,.\,(a + d)\,9 = (9 - 6d)(9 - 5d)\,9$

$= 9\,(30d^2 - 99d + 81) = 27\,(10d^2 - 33d + 27)$

$$\text{Min value at } d = \frac{-(-33)}{2\cdot 10} = \frac{33}{20}$$

Sol. 7 (B)

$x^2 - |x + 2| + x > 0$

Case - I $\quad x \geqslant -2 \Rightarrow x^2 - x - 2 + x > 0$

$\Rightarrow x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$\therefore x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Case - II $\quad x < -2$

$\Rightarrow x^2 + x + 2 + x > 0$

$\Rightarrow x^2 + 2x + 2 > 0$

$\Rightarrow x \in R \quad (\because D < 0)$

$\therefore x \in (-\infty, -2)$

$x \in$ (Case - I) $\cup$ (Case - II)

$\Rightarrow x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Ailter $|x + 2| < x^2 + x \Rightarrow -(x^2 + x) < x + 2 < x^2 + x$

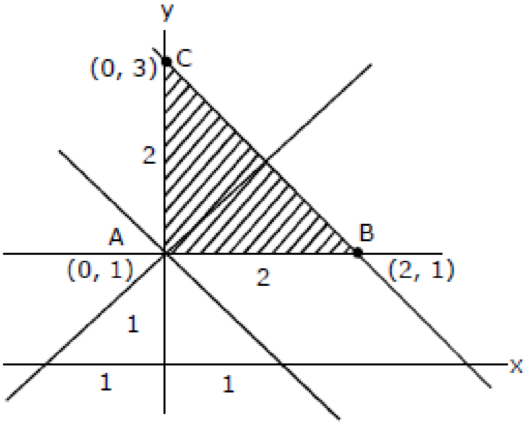
Sol. 8 (A)

$x^2 - y^2 + 2y - 1 = 0$

$x^2 (y - 1)^2 = 0$

$(x + y - 1)(x - y + 1) = 0$

$x + y = 1 \quad \& \; x - y + 1$



angle bisector are

$y = 1 \;\&\; x = 0$

A (0, 1), B(2, 1), C(0, 3)

area $\Delta ABC = \frac{1}{2} \cdot 2 \cdot 2 = 2$ sq. units

Sol. 9 (B)

$$0 < \cos\phi = \frac{1}{3} < \frac{1}{2} \;\&\; \theta = \frac{\pi}{6}$$

$$\Rightarrow \cos\frac{\pi}{2} < \cos\phi < \cos\frac{\pi}{6}$$

$$\Rightarrow \frac{\pi}{2} > \phi > \frac{\pi}{3} \Rightarrow \frac{\pi}{3} < \phi < \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{3} + \frac{\pi}{6} < \phi + \theta < \frac{\pi}{2} + \frac{\pi}{6} \Rightarrow \frac{\pi}{2} < \phi + \theta < \frac{2\pi}{3}$$

Sol. 10 (A)

Hyp. $xy - 3x - 2y = 0$

$f(x, y) = xy - 3x - 2y$

$$\frac{\delta f}{\delta x} = 0 \Rightarrow y = 3$$

$$\frac{\delta f}{\delta y} = 0 \Rightarrow x = 2 \qquad \text{Centre } (2, 3)$$

Asy. $xy - 3x - 2y + C = 0$

will pass through (2, 3)

$C = 6$

$xy - 3x - 2y + 6 = 0$

$(y - 3)(x - 2) = 0$

$x - 2 = 0,\; y - 3 = 0$

Sol. 11 (A)

Given, $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$, then

$$f\left(\frac{2x}{1+x^2}\right) = \log_e\left(\frac{1 - \frac{2x}{1+x^2}}{1 + \frac{2x}{1+x^2}}\right) \qquad \left[\because \left|\frac{2x}{1+x^2}\right| < 1\right]$$

$$= \log_e\left(\frac{\frac{1+x^2-2x}{1+x^2}}{\frac{1+x^2+2x}{1+x^2}}\right)$$

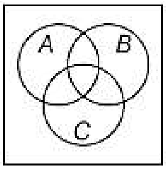
$$= \log_e\left(\frac{(1-x)^2}{(1+x)^2}\right) = \log_e\left(\frac{1-x}{1+x}\right)^2$$

$$= 2\log_e\left(\frac{1-x}{1+x}\right) \quad [\because \log_e|A|^m = m\log_e|A|]$$

$$= 2f(x) \qquad \left[\because f(x) = \log_e\left(\frac{1-x}{1+x}\right)\right]$$

Sol. 12 ( )

Exp. (c)



Let A be the set of even numbered students

then $n(A) = \left[\frac{140}{2}\right] = 70$

([.] denotes greatest integer function)

Let B be the set of those students whose number is divisible by 3,

then $n(B) = \left[\frac{140}{3}\right] = 46$

([.] denotes greatest integer function)

Let C be the set of those students whose number is divisible by 5,

then $n(C) = \left[\frac{140}{5}\right] = 28$

([.] denotes greatest integer function)

Now, $n(A \cap B) = \left[\frac{140}{6}\right] = 23$

(numbers divisible by both 2 and 3)

$n(B \cap C) = \left[\frac{140}{15}\right] = 9$

(numbers divisible by both 3 and 5)

$n(C \cap A) = \left[\frac{140}{10}\right] = 14$

(numbers divisible by both 2 and 5)

$n(A \cap B \cap C) = \left[\frac{140}{30}\right] = 4$

(numbers divisible by 2, 3 and 5)

and $n(A \cup B \cup C)$

$= \Sigma n(A) - \Sigma n(A \cap B) + n(A \cap B \cap C)$

$= (70 + 46 + 28) - (23 + 9 + 14) + 4 = 102$

$\therefore$ Number of students who did not opt any of the three courses

$=$ Total students $- n(A \cup B \cup C) = 140 - 102 = 38$

Sol. 13 (C)

Given $f(x) = x^3 + 5x + 1$

Now, $f'(x) = 3x^2 + 5 > 0, \ \forall \ x \in R$

Thus, $f(x)$ is strictly increasing function.

So, $f(x)$ is one-one function.

Clearly, $f(x)$ is a continuous function and also increasing on R.

$\therefore \lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} = \infty$

Hence, $f(x)$ takes every value between $-\infty$ and ∞.

Thus, $f(x)$ is onto function.

Sol. 14 (A)

Let $W = \{CAT, TOY, YOU, \ldots\}$

Clearly, R is reflexive and symmetric but not transitive. $[\because {}_{CAT}R_{TOY}, {}_{TOY}R_{YOU} \not\Rightarrow {}_{CAT}R_{YOU}]$

Sol. 15 (D)

Since, for every elements of A, there exists elements $(3, 3), (6, 6), (9, 9), (12, 12) \in R \Rightarrow R$ is reflexive relation.

Now, $(6, 12) \in R$ but $(12, 6) \notin R$, so it is not a symmetric relation.

Also, $(3, 6), (6, 12) \in R \Rightarrow (3, 12) \in R$

$\therefore R$ is transitive relation.

Sol. 16 (D)

Given, function $f(x) = a^x, a > 0$ is written as sum of an even and odd functions $f_1(x)$ and $f_2(x)$ respectively.

Clearly, $f_1(x) = \frac{a^x + a^{-x}}{2}$ and $f_2(x) = \frac{a^x - a^{-x}}{2}$

So, $f_1(x + y) + f_1(x - y)$

$$= \frac{1}{2}[a^{x+y} + a^{-(x+y)}] + \frac{1}{2}[a^{x-y} + a^{-(x-y)}]$$

$$= \frac{1}{2}\left[a^x a^y + \frac{1}{a^x a^y} + \frac{a^x}{a^y} + \frac{a^y}{a^x}\right]$$

$$= \frac{1}{2}\left[a^x\left(a^y + \frac{1}{a^y}\right) + \frac{1}{a^x}\left(\frac{1}{a^y} + a^y\right)\right]$$

$$= \frac{1}{2}\left(a^x + \frac{1}{a^x}\right)\left(a^y + \frac{1}{a^y}\right)$$

$$= 2\left(\frac{a^x + a^{-x}}{2}\right)\left(\frac{a^y + a^{-y}}{2}\right) = 2f_1(x) \cdot f_1(y)$$

SOL. 17 (C)

Given function $f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$

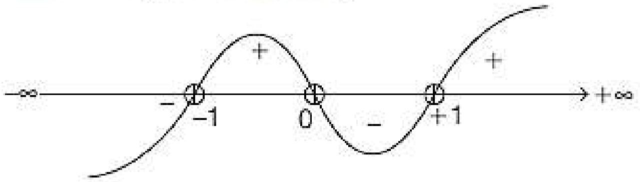
For domain of $f(x)$

$$4 - x^2 \neq 0 \Rightarrow x \neq \pm 2 \quad \ldots(i)$$

and $x^3 - x > 0$

$\Rightarrow x(x-1)(x+1) > 0$

From Wavy curve method,

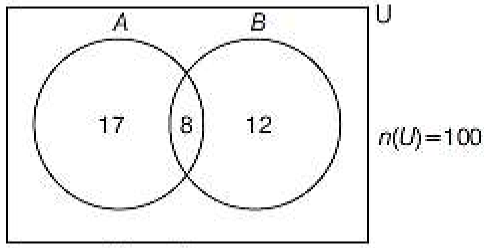


From Eqs. (i) and (ii), we get the domain of $f(x)$ as $(-1, 0) \cup (1, 2) \cup (2, \infty)$.

Sol . 18 (D)

Let the population of city is 100.

Then, $n(A) = 25$, $n(B) = 20$ and $n(A \cap B) = 8$



Venn diagram

So, $n(A \cap \overline{B}) = 17$ and $n(\overline{A} \cap B) = 12$

According to the question, Percentage of the population who look into advertisement is

$$= \left[\frac{30}{100} \times n(A \cap \overline{B})\right] + \left[\frac{40}{100} \times n(\overline{A} \cap B)\right] + \left[\frac{50}{100} \times n(A \cap B)\right]$$

$$= \left(\frac{30}{100} \times 17\right) + \left(\frac{40}{100} \times 12\right) + \left(\frac{50}{100} \times 8\right)$$

$$= 5.1 + 4.8 + 4 = 13.9$$

Sol. 19 (A)

According to given information, we have if $k \in \{4, 8, 12, 16, 20\}$

Then, $f(k) \in \{3, 6, 9, 12, 15, 18\}$

[$\because$ codomain $(f) = \{1, 2, 3, \ldots, 20\}$]

Now, we need to assign the value of $f(k)$ for $k \in \{4, 8, 12, 16, 20\}$ this can be done in ${}^6C_5 \cdot 5!$ ways $= 6 \cdot 5! = 6!$ and remaining 15 element can be associated by $15!$ ways.

$\therefore$ Total number of onto functions $= \lfloor 15 \lfloor 16 = 15!\, 6!$

SOL. 20 (B)

Given A set $X = \{1, 2, 3, 4, 5\}$

To find The number of different ordered pairs (Y, Z) such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z = \phi$.

Since, $Y \subseteq X$, $Z \subseteq X$, hence we can only use the elements of X to construct sets Y and Z.

Method 1

$n(Y)$	Number of ways to make Y	Number of ways to make Z such that $Y \cap Z = \phi$
0	${}^5C_0$	2^5
1	${}^5C_1$	2^4
2	${}^5C_2$	2^3
3	${}^5C_3$	2^2
4	${}^5C_4$	2^1
5	${}^5C_5$	2^0

Let us explain anyone of the above 6 rows say third row. In third row,

Number of elements in $Y = 2$

$\therefore$ Number of ways to select $Y = {}^5C_2$ ways

Because any 2 elements of X can be part of Y.

Now, if Y contains any 2 elements, then these 2 elements cannot be used in any way to construct Z, because we want $Y \cap Z = \phi$.

And from the remaining 3 elements which are not present in Y, 2^3 subsets can be made each of which can be equal to Z and still $Y \cap Z = \phi$ will be true.

Hence, total number of ways to construct sets Y and Z such that $Y \cap Z = \phi$

$$= {}^5C_0 \times 2^5 + {}^5C_1 \times 2^{5-1} + \ldots + {}^5C_5 \times 2^{5-5}$$

$$= (2 + 1)^5 = 3^5$$

PHYSICS

Sol. 21

34. (c) : Induced emf $|\varepsilon| = L\frac{dI}{dt}$

where L is the self inductance and $\frac{dI}{dt}$ is the rate of change of current.

$\therefore$ Dimensional formula of

$$L = \frac{|\varepsilon|}{\frac{dI}{dt}} = \frac{[ML^2T^{-3}A^{-1}]}{[AT^{-1}]} = [ML^2T^{-2}A^{-2}]$$

Sol. 22

(c) : Units of $b = \frac{x}{t^2} = \frac{km}{s^2}$

Sol. 23

$$\text{Total time taken} = \frac{100}{40} + \frac{100}{v}$$

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$48 = \frac{200}{\left(\frac{100}{40} + \frac{100}{v}\right)} \text{ or } 48 = \frac{2}{\left(\frac{1}{40} + \frac{1}{v}\right)}$$

$$\text{or } \frac{1}{40} + \frac{1}{v} = \frac{1}{24} \text{ or } \frac{1}{v} = \frac{1}{24} - \frac{1}{40} = \frac{5-3}{120} = \frac{1}{60}$$

or $v = 60$ km/h

Sol. 24

(d) : Because the slope is highest at C,

$v = \frac{ds}{dt}$ is maximum

Sol. 25 (b) : In one dimensional motion, the body can have one value of velocity at a time but not two values of velocities at a time.

Sol. 26 (D)

Action = Energy × Time

= $M^1L^2T^{-2} \times T^1$

= $M^1L^2T^{-1}$

Sol. 27 (B)

Fundamental quantities does not depends each other So, in length, time and velocity here velocity is derived quantities.

Sol. 28 (C)

The magnitude of acceleration is constant in (A) and decreasing in (B)

In (A) $\rightarrow$ r constant, $a_t = 0$;

v constant, $a_r = \frac{V^2}{R}$ constant

In (B) $\rightarrow$ r is increasing, V constant

$a_t = 0$; $a_r = \frac{V^2}{R}$ decreasing

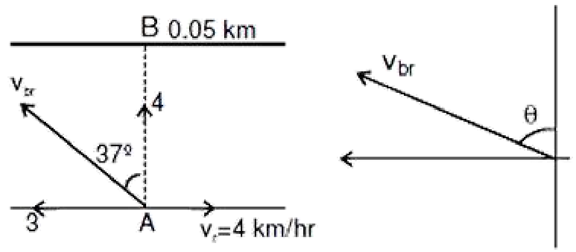
Sol. 29 (A)

If v = 0 or v = constant then frame is inertial.

Sol. 30 (B) (13/5) P

Sol. 31 (A) Displacement from the mean position

Sol. 32 (B)



$V_{br} = 5$ km/hr

$\sin\theta = \frac{v_r}{5}$

$t = \frac{d}{v_{br}\cos\theta} \Rightarrow \cos\theta = \frac{4}{5} \Rightarrow \theta = 37º$

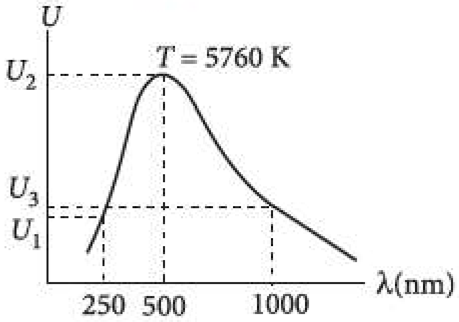
$\sin 37º = \frac{v_r}{5} \Rightarrow v_r = 3\text{km/hr}$

Sol. 33 (D)

Sol. 34 (A) 1: 2

Sol. 35 (B) : According to Wein's displacement law

$$\lambda_m = \frac{b}{T} = \frac{2.88\times10^6 \text{ nm K}}{5760 \text{ K}} = 500 \text{ nm}$$



Sol. 36 (C)

Let $d_{\min}$ = minimum diameter of brass.

Then, stress in brass rod is given by

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_{\min}^2} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

For stress not to exceed elastic limit, we have

$\sigma \le 379$ MPa

$$\Rightarrow \quad \frac{4F}{\pi d^2_{\min}} \le 379\times10^6$$

Here, $F = 400\,\text{N}$

$$\therefore \quad d^2_{\min} = \frac{1600}{\pi\times379\times10^6}$$

$$\Rightarrow \quad d_{\min} = 1.16\times10^{-3} \text{ m} = 1.16\,\text{mm}$$

Sol.37 (D)

Let mass of given body is m. Then, it's weight on earth's surface $= mg_e$

where, g_e = acceleration due to gravity on earth's surface

and weight on the surface of planet $= mg_p$

g_p = acceleration due to gravity on planet's surface.

Given,

$$\frac{mg_e}{mg_p} = \frac{9}{4} \Rightarrow \frac{g_e}{g_p} = \frac{9}{4}$$

But $g = \dfrac{GM}{R^2}$, so we have

$$\frac{\left(\dfrac{GM}{R^2}\right)}{\left(\dfrac{GM_p}{R_p^2}\right)} = \frac{9}{4}$$

where, M = mass of earth,

R = radius of earth,

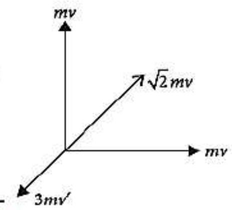
M_p = mass of plane $= \dfrac{M}{9}$ (given)

and R_p = radius of planet.

$$\Rightarrow \quad \frac{M}{M_p}\cdot\frac{R_p^2}{R^2} = \frac{9}{4} \Rightarrow 9\cdot\left(\frac{R_p}{R}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \quad \frac{R_p}{R} = \frac{1}{2} \Rightarrow R_p = \frac{R}{2}$$

Sol.38 (D)



Let the speed of the third fragment of mass $3m$ be v'. From law of conservation of linear momentum,

$$3mv' = \sqrt{2}\,mv \Rightarrow v' = \frac{\sqrt{2}\,v}{3} \qquad \text{...(i)}$$

$\therefore$ Energy released during the process is,

$$\text{K.E.} = 2\left(\frac{1}{2}mv^2\right) + \frac{1}{2}(3m)v'^2 = mv^2 + \frac{1}{2}(3m)\frac{2v^2}{9}$$

(Using eqn. (i))

$$= mv^2 + \frac{mv^2}{3} = \frac{4}{3}mv^2$$

Sol. 39 (D**)**

According to question, velocity of unit mass varies as

$v(x) = \beta x^{-2n}$ ...(i)

$\frac{dv}{dx} = -2n\beta x^{-2n-1}$ ...(ii)

Acceleration of the particle is given by

$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v$

Using equation (i) and (ii), we get

$a = (-2n\beta x^{-2n-1}) \times (\beta x^{-2n}) = -2n\beta^2 x^{-4n-1}$

sol. 40 (B) decrease

Chemistry

Sol. 46 (A)

	N_2	+	$3H_2$	$\rightarrow$	$2NH_3$
	1 mole		3 mole		2 mole
or	1/2 mole		3/2 mole		1 mole
or	$10 \times 1/2 = 5$		$10 \times 3/2 = 15$		$10 \times 1 = 10$

As only 50% ammonia formation is expected so composition of gaseous mixture under the above mentioned condition is as follows:

$H_2 = 30 - 15 = 15$ L

$N_2 = 30 - 5 = 25$ L

$NH_3 = 10$ L

Sol. 47 (C) 1 mole of electrons weighs 0.54 mg

Sol. 48 (C) Ni^{2+}, Ti^{3+}

Sol. 49 (A) B < Be < C < O < N

Sol. 50 (A) Three

Sol. 51 (C) 1/8

Sol. 52 (A) but-3-enoic acid

Sol. 53 (C) 3-bromo-1-chlorocyclohexene

Sol. 54 (D) B_2H_6

Sol. 55 (B)

$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$

Therefore, dimensions of pressure $= \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

and dimensions of energy per unit volume

$= \frac{\text{Energy}}{\text{Volume}} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

Sol. 56 (A) 1 and 2

Sol. 57 (C)

$3C + 2Al_2O_3 \longrightarrow 4Al + 3CO_2$
(From bauxite)

4 moles of Al is produced by 3 moles of C.

1 mole of Al is produced by $\frac{3}{4}$ mole of C.

$\frac{270 \times 1000}{27} = 10^4$ moles of Al is produced by $\frac{3}{4} \times 10^4$ moles of C.

Amount of carbon used $= \frac{3}{4} \times 10^4 \times 12$ g

$= \frac{3}{4} \times 10 \times 12$ kg $= 90$ kg

Sol . 58(A)

: Species having same no. of electrons are called isoelectronic species.

The no. of electrons in $CO = CN^- = NO^+ = C_2^{2-} = 14$. So, these are isoelectronic species.

Sol. 59 (D)

Unnilunium – Mendelevium $\Rightarrow$ (a)-(i)

Unniltrium – Lawrencium $\Rightarrow$ (b)-(ii)

Unnilhexium – Seaborgium $\Rightarrow$ (c)-(iii)

Unununnium – Roentgenium $\Rightarrow$ (d)✕(iv)

Sol. 60 (C) Along the period, as we move from

Li → Be → B → C, the electronegativity increases

and hence the EN difference between the element and

Cl decreases and accordingly, the covalent character

increases. Thus LiCl < BeC_{l2} < BC_{l3} < CC_{l4} is the correct

order of covalent bond character.